

§7 MOMENTUM

PROBLEM 1.

A particle of mass m moving with velocity $\mathbf{v}_1$ leaves a half-space in which its potential energy is a constant U_1 and enters another in which its potential energy is a different constant U_2 . Determine the change in the direction of motion of the particle.

SOLUTION: The potential energy only depend on the co-ordinate perpendicular to the plane separating the half-space. Thus, the component of the momentum in that plane is conserved. Denoting by θ_1 and θ_2 the angles between the normal to the plane and the velocities v_1 and v_2 of the particle before and after passing the plane, we have

$$\begin{aligned} P_1 \sin \theta_1 &= P_2 \sin \theta_2 \\ \Rightarrow v_1 \sin \theta_1 &= v_2 \sin \theta_2 \\ \Rightarrow \frac{\sin \theta_1}{\sin \theta_2} &= \frac{v_2}{v_1}. \end{aligned} \quad (2.1)$$

The potential energy of the system is also independent of time, therefore the energy of the particle is conserved. Posing $E_1 = T_1 + U_1$ and $E_2 = T_2 + U_2$ as the energy of the particle before and after passing the plane, the law of conservation of energy requires

$$\begin{aligned} E_1 &= E_2 \\ \Rightarrow T_1 + U_1 &= T_2 + U_2 \\ \Rightarrow \frac{1}{2}mv_1^2 + U_1 &= \frac{1}{2}mv_2^2 + U_2 \\ \Rightarrow v_2^2 &= v_1^2 + \frac{2}{m}(U_1 - U_2). \end{aligned} \quad (2.2)$$

By substituting Eq. (2.2) in the square of Eq. (2.1), we get

$$\left(\frac{\sin \theta_1}{\sin \theta_2} \right)^2 = \frac{v_1^2 + \frac{2}{m}(U_1 - U_2)}{v_1^2}.$$

After taking the square root, the result is

ANSWER TO PROBLEM 1.

$$\frac{\sin \theta_1}{\sin \theta_2} = \sqrt{1 + \frac{2}{mv_1^2}(U_1 - U_2)}$$