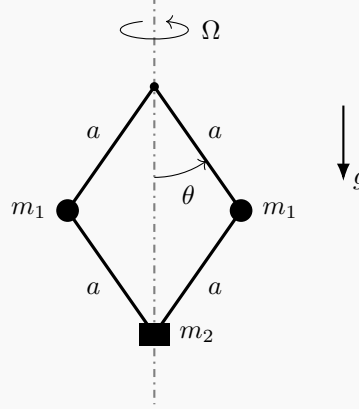


PROBLEM 4.

Find the Lagrangian of a simple pendulum of the system below when placed in a uniform gravitational field (acceleration g). The particle m_2 moves on a vertical axis and the whole system rotates about this axis with a constant angular velocity Ω .



SOLUTION: The position of each particle m_1 is best described in cylindrical coordinates, which is

$$\begin{aligned} r_1 &= a \sin \theta \\ \phi_1 &= \phi \\ z_1 &= a \cos \theta, \end{aligned}$$

where ϕ is the angle of rotation of the system about the axis; $\dot{\phi} = \Omega$. The kinetic energy of each particle m_1 is thus

$$\begin{aligned} T_1 &= \frac{1}{2} m_1 (\dot{r}_1^2 + r_1^2 \dot{\phi}_1^2 + \dot{z}_1^2) \\ &= \frac{1}{2} m_1 (a^2 \dot{\theta}^2 \cos^2 \theta + a^2 \Omega^2 \sin^2 \theta + a^2 \dot{\theta}^2 \sin^2 \theta) \\ &= \frac{1}{2} m_1 a^2 (\dot{\theta}^2 + \Omega^2 \sin^2 \theta). \end{aligned} \quad (1.5)$$

The potential energy of this particle can be found by using the z component of his position, namely

$$\begin{aligned} V_1 &= -m_1 g z_1 \\ &= -m_1 g a \cos \theta. \end{aligned} \quad (1.6)$$

The particle m_2 , in its case, can only move up and down, thus its position can be completely defined by

$$z_2 = 2a \cos \theta.$$

Then, the kinetic energy of each particle m_2 is

$$\begin{aligned} T_2 &= \frac{1}{2} m_2 \dot{z}_2^2 \\ &= m_2 a^2 \dot{\theta}^2 \sin^2 \theta \end{aligned} \quad (1.7)$$

and the potential energy is

$$\begin{aligned} V_2 &= -m_2 g z_2 \\ &= -2m_2 g a \cos \theta. \end{aligned} \quad (1.8)$$

Using Eq. (1.5), Eq. (1.6), Eq. (1.7) and Eq. (1.8), the Lagrangian of the system is

$$L = 2(T_1 + T_2 - V_1 - V_2),$$

therefore

ANSWER TO PROBLEM 4.

$$L = m_1 a^2 (\dot{\theta}^2 + \Omega^2 \sin^2 \theta) + 2m_2 a^2 \dot{\theta}^2 \sin^2 \theta + 2(m_1 + 2m_2) g a \cos \theta$$