

PROBLEM 2.

Express the effective cross-section (§18 Prob. 1) as a function of the energy ϵ lost by a scattered particle.

SOLUTION: The energy loss by a particle of mass m_1 is equal to the energy gained by the sphere of mass m_2 . Knowing that the sphere is initially at rest, and using v'_2 from §17 Prob. 1, the energy gained is

$$\epsilon = E'_2 = \frac{1}{2}m_2v'^2_2 = \frac{2m_1^2m_2}{(m_1 + m_2)^2}v_\infty^2 \sin^2 \frac{1}{2}\chi = \epsilon_{max} \sin^2 \frac{1}{2}\chi.$$

Using the double angle identity, the variation of ϵ is

$$d\epsilon = \epsilon_{max} \sin \frac{1}{2}\chi \cos \frac{1}{2}\chi d\chi = \frac{1}{2}\epsilon_{max} \sin \chi d\chi.$$

Using Eq. (4.4) from §18 Prob. 1, the effective cross section can thus be express as

ANSWER TO PROBLEM 2.

$$d\sigma = \frac{\pi a^2 d\epsilon}{\epsilon_{max}}$$