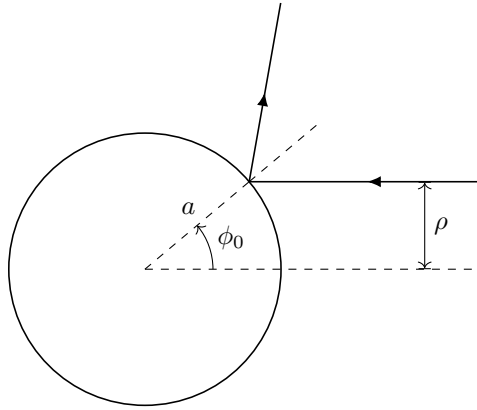


§18 SCATTERING

PROBLEM 1.

Determine the effective cross-section for scattering of particles from a perfectly rigid sphere of radius a (*i.e.* when the interaction is such that $U = \infty$ for $r < a$ and $U = 0$ for $r > a$).

SOLUTION: The particle is reflected from the sphere at the same angle ϕ_0 to the normal (*i.e.* the radius) as it is incident, as shown in the figure.



From the figure, we can see

$$\rho = a \sin \phi_0 = a \sin \frac{1}{2}(\pi - \chi) = a \cos \frac{1}{2}\chi. \quad (4.3)$$

Substituting Eq. (4.3) in formula (18.7) or (18.8) of the book, we have

$$d\sigma = \frac{1}{2}\pi a^2 \sin \chi d\chi = \frac{1}{4}a^2 d\chi. \quad (4.4)$$

If we integrate $d\sigma$ over all angles, we find

$$\sigma = \frac{1}{2}\pi a^2 \int_{-\pi/2}^{\pi/2} \sin \chi d\chi = \pi a^2,$$

which is the projection of the sphere perpendicular to the beam (*i.e.* the impact area). We need to express Eq. (4.4) in the L system using formula (17.4) of the book. If this equation is solved for $\cos \theta_1$, we get, as in §16 Prob. 2 with V/v_0 replaced by m_1/m_2 ,

$$\begin{aligned}
\tan \theta_1 &= \frac{m_2 \sin \chi}{m_1 + m_2 \cos \chi} \\
\Rightarrow \tan^2 \theta_1 \left(\frac{m_1}{m_2} + \cos \chi \right)^2 &= 1 - \cos^2 \chi \\
\Rightarrow \cos \chi &= -\frac{m_1}{m_2} \sin^2 \theta_1 \pm \cos \theta_1 \sqrt{1 - \frac{m_1^2}{m_2^2} \sin^2 \theta_1} \\
\Rightarrow \frac{1}{2} \sin \chi d\chi &= \frac{1}{2} \sin \theta_1 d\theta_1 \left(2 \frac{m_1}{m_2} \cos \theta_1 \pm \frac{1 + (m_1^2/m_2^2) \cos 2\theta_1}{\sqrt{1 - (m_1^2/m_2^2) \sin^2 \theta_1}} \right).
\end{aligned}$$

We thus get

ANSWER TO PROBLEM 1.

$$\begin{aligned}
d\sigma_1 &= \frac{1}{4} a^2 \left(2 \frac{m_1}{m_2} \cos \theta_1 \pm \frac{1 + (m_1^2/m_2^2) \cos 2\theta_1}{\sqrt{1 - (m_1^2/m_2^2) \sin^2 \theta_1}} \right) d\theta_1 \\
d\sigma_2 &= a^2 |\cos \theta_2| d\theta_2.
\end{aligned}$$