

§16 DISINTEGRATION OF PARTICLES

PROBLEM 1.

Find the relation between the angles θ_1, θ_2 (in the L system) after a disintegration into two particles.

SOLUTION: The angles of the particles, in the C system, are related by

$$\theta_0 = \theta_{10} = \pi - \theta_{20},$$

where θ_0 as been defined to simplify the notation. From formula (16.5) of the book, we have

$$\begin{aligned} V + v_{10} \cos \theta_0 &= v_{10} \sin \theta_0 \cot \theta_1 \\ V - v_{20} \cos \theta_0 &= v_{20} \sin \theta_0 \cot \theta_2. \end{aligned}$$

We must eliminate θ_0 from the two equations above. To do so, we can first solve for $\cos \theta_0$ and $\sin \theta_0$ which give us

$$\sin \theta_0 = \frac{V + v_{10} \cos \theta_0}{v_{10} \cot \theta_1} = \frac{V - v_{20} \cos \theta_0}{v_{20} \cot \theta_2} \quad (4.1)$$

$$\begin{aligned} \Rightarrow V v_{20} \cot \theta_2 + v_{10} v_{20} \cos \theta_0 \cot \theta_2 &= V v_{10} \cot \theta_1 - v_{10} v_{20} \cos \theta_0 \cot \theta_1 \\ \Rightarrow \cos \theta_0 &= \frac{V (v_{10} \cot \theta_1 - v_{20} \cot \theta_2)}{v_{10} v_{20} (\cot \theta_1 + \cot \theta_2)}. \end{aligned} \quad (4.2)$$

Using the sum of square of Eq. (4.1) and Eq. (4.2), we get

$$\begin{aligned} \left(\frac{V + v_{10} \cos \theta_0}{v_{10} \cot \theta_1} \right)^2 + \left(\frac{V (v_{10} \cot \theta_1 - v_{20} \cot \theta_2)}{v_{10} v_{20} (\cot \theta_1 + \cot \theta_2)} \right)^2 &= 1 \\ \Rightarrow (v_{10} + v_{20})^2 + (v_{10} \cot \theta_1 - v_{20} \cot \theta_2)^2 &= \frac{v_{10}^2 v_{20}^2}{V^2} (\cot \theta_1 + \cot \theta_2)^2 \\ \Rightarrow v_{10}^2 \csc^2 \theta_1 + v_{20}^2 \csc^2 \theta_2 - 2 v_{10} v_{20} \cot \theta_1 \cot \theta_2 &= \frac{v_{10}^2 v_{20}^2 \sin^2 (\theta_1 + \theta_2)}{V^2 \sin^2 \theta_1 \sin^2 \theta_2} \\ \Rightarrow v_{20}^2 \sin^2 \theta_1 + v_{10}^2 \sin^2 \theta_2 - 2 v_{10} v_{20} \sin \theta_1 \sin \theta_2 \cos \theta_1 \cos \theta_2 &= \frac{v_{10}^2 v_{20}^2}{V^2} \sin^2 (\theta_1 + \theta_2). \end{aligned}$$

Using formula (16.2) of the book and the relation $v_{10}/v_{20} = m_2/m_1$, we obtain

ANSWER TO PROBLEM 1.

$$\frac{m_2}{m_1} \sin^2 \theta_2 + \frac{m_1}{m_2} \sin^2 \theta_1 - 2 \sin \theta_1 \sin \theta_2 \cos \theta_1 \cos \theta_2 = \frac{2\epsilon}{(m_1 + m_2)V^2} \sin^2 (\theta_1 + \theta_2).$$